

KINEMATIC ANALYSIS OF PLANAR WHOLE ARM MANIPULATOR

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Abstract A whole arm manipulator employs all the available surfaces on all its moving links to act upon objects to grasp and / or manipulate an object. The forward kinematic analysis is carried out for a given manipulator with given object dimensions. Input joint velocities are used to animate the manipulator with one, two or three point contacts. The animation demonstrates both the prehensile and non-prehensile manipulation. The inverse problem classifies the type of contacts as no contact, non tangential contact or tangential contact at one or more points.

Keywords: Contact point, Non-prehensile manipulation, Grasp

INTRODUCTION

The human hand is versatile in its interactions with the environment. The important functions of human hand are to explore, to restrain and move the objects. To restrain and move objects which are large man uses his arms and chest, hands for medium sized objects and the fingers for smaller objects. The motion of the grasped object can range from small precision motion to large motion. [Iberall, 1987] using the oppositions classifies the nature of human prehensile grasping into three types, namely, pad opposition, palm opposition and side opposition and gives comparison of taxonomical classifications. [Skinner, 1975] designs a multiple prehension manipulator. The robotic devices intended to exploit the capabilities of human hand have been pioneered by MIT whole arm manipulator project [Salisbury, 1987], [Salisbury *et al.*, 1988]. In Whole Arm Manipulation (WAM), the intermediate links of the arm of the manipulator can be used to act upon objects by imparting forces and motions and by imposing constraints. There are two ways to manipulate the object, namely, 1)grasp the object and carry it to the new location and disengage it (prehensile manipulation), or 2)push the object if the initial and final goal locations share the same support surface(non-prehensile manipulation).

Cai and Roth, (1986) studied the contact kinematics of planar rigid bodies in point contact. [Montana, 1988] derives the equations of kinematic contact which involve curvature of contacting bodies. [Cutkosky, 1989] studied the grasp models and their choice in design of hands for manufacturing tasks by the machinists. [Trinkle *et al.*, 1987], [Reynaerts and Brussel, 1993], [Yashima and Yamaguchi, 1999] describe the kinematics of envelope manipulation. [Cole

et al., 1988], [Harada *et al.*, 2000] consider rolling contact point during manipulation. [Nagashima *et al.*, 1997] deal with manipulation using sliding contact point. The velocity analysis of two 3-R robots manipulating a disk on third link is presented by [Pennock, and Squires, 1998]. [Vassura and Bicchi, 1989], [Bicchi and Melchiorri, 1992], [Bicchi *et al.*, 1995] address the issues involved in mobility and manipulability of multiple limb robots. [Omata and Sekiyama, 1997] studied transition from enveloping grasp to finger tip grasping. [Salisbury, 1987], [Shimajima *et al.*, 1987], [Tischler *et al.*, 1998] deal with manipulator kinematic synthesis. Pushing is a non-prehensile manipulation. [Mason, 1985] pioneered the study of the mechanics of pushing. [Erdmann, 1998] studied nonprehensile manipulation with two palms. [Mason, 1999] reviews the progress in nonprehensile manipulation. A review of the issues involved in WAM has been carried out by the authors [Pilli and Mruthyunjaya, 1999]. [Bicchi and Kumar, 2001] present a survey on the state of art of robotic grasping and manipulation.

This paper carries out analysis of planar WAM manipulating a circular object in terms of forward kinematics, and inverse kinematics. The number of links in the manipulator varies from 1 to 4.

ANALYSIS

It is assumed that the links and the object are rigid. Links are straight and joints are revolute. The object is circular. Each link can have at most one point of contact with the object at any given time. The interface between the object with manipulator and the support frame is assumed to be frictionless.

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Direct Kinematics

The direct kinematic analysis concerns with animation of the manipulation given the initial pose with angular velocities at the joints of the manipulator as the inputs. Finding the initial pose is an inverse problem. The initial pose is determined from the location information for the given manipulator and the object. In case of manipulator with more than one link, the link contacting the object can be left or right tangential, while any of the previous links can be left / right tangential or posing towards the center of the object. For a tangential contact, from geometry the necessary and sufficient conditions are given by eqns.(1), (2), and (3).

$$l_{con} = \sqrt{d^2 - (r+th)^2} \tag{1}$$

$$0 \leq l_{con} \leq l \tag{2}$$

$$l_{con} = l_{con} * \cos(\tan^{-1}(th/d)) \tag{3}$$

where r - radius of the object,
 l - length of the link under consideration,
 th - half width of the link,
 d - distance between link base and object center,
 l_{con} - distance between contact point and link base.
 l_{con} - projection of l_{con} along the center line of the link.

Single-link manipulator The contact configuration is shown in fig.1. In a small interval of time ‘dt’ the link moves through $(\omega_1 dt)$ and the object experiences a displacement $(V_E dt)$ in the direction of velocity V_E . With this incremental displacement the object is moved away along the link, which is rotating with constant angular velocity. The trajectory turns out to be an Archemedian spiral [Tuma, 1987]. A change in sense of angular velocity leads to breaking of the contact and no motion of the object is possible. The manipulation is non-prehensile.

Two-link manipulator The contact can be one point contact on either of the links or two point contact with both the links. For manipulator with equal link lengths shown in fig.2, the limiting normalized effective contact length and the maximum size of a cylindrical object that can be grasped depend on the slenderness ratio given by eqns.(4) and (5).

$$l/l = 1 - th(l * \tan(\theta/2)) \tag{4}$$

$$r/l = \tan((\theta/2)) - th/l \tag{5}$$

Three-link manipulator The contact can be one point contact on any one of the links, or two points contact on any of the two link combinations or three points contact with all the three links.

The position equations of contact point are used to obtain the velocity. Analysis of a dyad shown in fig.3 encompasses majority of situations arising in two point contacts. Here, for the purpose of analysis, we actuate one link at a time and then superimpose the two results

to get the final displacement of the object. Let us consider actuating link 1 (fig.3.a). Link 2, which now behaves as a rigid body and manipulator can be considered as a single link rotating with points of contact at points ‘E’ and ‘F’ with angular velocity (ω_1) . The two velocities at contact points are transferred to the object. The object undergoes an instantaneous rotation about the base of the manipulator, which is the instantaneous center of velocity. Next consider actuating only the link 2 (fig.3.b). For maintaining contact the second link should move in clock-wise sense. Here, object is constrained to slide along link 1. In time ‘dt’ as link rotates through $(\omega_2 dt)$ the object is displaced through $(V_E dt)$ along link one. When the two joints are actuated simultaneously, we superimpose these two velocities.

When link 1 and link 2 contact the object, the manipulator behaves as a dyad. In case of link 2 and link 3 contacting the object, link 1 moves with angular velocity (ω_1) , the object also moves with same angular velocity about end point of link 1. The influence of second and third link actuation remains same as discussed for the dyad. The velocity of point A and velocity of dyad are combined to obtain the velocity of the object. When the contact is on link 1 and link 3 and link 1 is not actuated, the contact configuration is shown in fig.4. From geometry the velocity of contact point is obtained. When only link 1 is actuated, with link 3 being held stationary, the analysis is similar to one point contact. Superimposing the two cases, we can compute the object velocity when both the links are actuated.

The three points of contact in the initial configuration are obtained by applying eqns.(1) to (3), successively to all the three links shown in fig.5 from the base. Here we can actuate link 1 only because actuation of link 2 and link 3 results in either crushing of the object or loss of contact at some of the contact points.

Inverse Kinematics

The task is to find the contact configurations for the given manipulator and the object size and their position. A configuration is a feasible solution if the links do not intersect amongst themselves and the object. The solution can be the possible points of contact based on the size and position of manipulator and the object or a point of interest may be specified on the object. In general, we can have any of the following types of contact namely: no-contact, non-tangential contact, tangential contact at one point, two points or three points. If the object is cutting into the base of the manipulator such a contact is classified as no contact.

No contact The object cannot be contacted when 1) the object is beyond reach, 2) the object is cutting into the base of the manipulator, 3) the object may be within reach but the point of interest may lie outside reachable zone, and 4) the point of interest may be contacted, but links may be self intersecting or intersecting with the object.

Non-tangential contact Let 'E' be the point of interest on the object shown in fig.6. We have to find the angles, which the links should make with the horizontal. We note that ∞^3 solutions are possible. The configuration is a 4-bar chain (PABE in fig.6). A check is carried out to ascertain whether the 4-bar chain is Grashoff or non-Grashoff type, and then the extreme positions of link 1 and angles between the links can be calculated. The non-tangential contact configurations at any specified point in the feasible region of contact can be obtained.

Tangential contact A tangential contact can occur over any of the links or their combinations. For one point contact, a check for tangential contact is carried out using eqns.(1) to (3) starting from the base. In case of single link manipulator, if contact is possible the link can be either left or right tangential to the object. In case of a manipulator with two links, if object is contacting non-tangentially or beyond reach of link 1 then its end point is taken as a pivot and a check is carried out for tangential contact on subsequent link(s). Once a tangential contact is established on a particular link the subsequent links are assumed to line up with the link contacting tangentially. A check is also carried out for the intersection of the object with the links and intersection amongst the links. In case of 2 point contact over any two links a total of 14 configurations are possible. In case of three points of contact, all the three links can contact the object in two different ways with all the links being either left or right tangential.

Also, an inverse solution is also carried out when a particular point on the object is of interest. Let 'E' be the point of interest. If the point of interest 'E' is to contact with link 2 (fig.7), draw a line passing through P and E. We have to locate the pivot point A on the tangent such that $PA = l_1$ and $abs(l_x) \leq l_2$. The location of point 'A' is the solution of a quadratic equation. The real solutions are valid, however a negative solution indicates the position of 'A' is on the other side. For a tangential contact on link 3 the object can contact anywhere along the link 3. There are two ways in which the link 3 can orient. Also, pivot 'A' has two possible positions as shown in fig.8.

Motion Classification and Grasp

With one point contact, the object moves away from the base. If the sense of input angular velocity of the link is changed the contact breaks and no motion is transferred to the object. In case of two or three points of contact, if any two points of contact are on either side of the line joining the base of the manipulator to the object center, bidirectional motion is possible otherwise, the contact breaks when the sense of angular velocity is reversed. With one or two points of contact only a non-prehensile manipulation is possible. With three points of contact, it is possible to constrain the object fully if the angle between two successive lines joining the contact point and the object center is less than 180 degrees. In such a case the 'grasp' is achieved and the manipulation

is prehensile. A grasp also indicates bidirectional motion.

RESULTS AND DISCUSSION

The simulation is carried out in MATLAB5.3. For a specified single link manipulator and object position, as the thickness of the manipulator or radius of the object increases the initial contact length is smaller and object can be manipulated over longer distance before the object goes beyond the reach of the manipulator. The manipulation is non-prehensile.

A three link manipulator manipulating the object with an initial one point of contact is shown in figs.9. With initial one point contact on link 1, the object moves away from the base, the trajectory is spiral until it comes in contact with link 2. With two points of contact the trajectory is more general and. With three points of contact only link 1 is given the input, hence the object moves along a circular path. However the manipulation shown in fig.9 is unidirectional. Figure 10 shows a prehensile manipulation wherein grasp is achieved as the contact points lie on either side of the line joining the base of the manipulator and the object center. The grasp is also achieved as the angle between the two successive lines joining the contact point and the object center is less than 180 degrees.

A 3-link manipulator with three points of contact can be actuated only at the base and the trajectory is along a circular arc. In order to manipulate the object along a more general trajectory four link manipulator with actuation at the base and end of link one is envisaged. The object is held by the last three links. The input may be at the base and / or at joint 1. The analysis is similar to two point contact on link 2 and link 3 of a 3-link manipulator. The manipulation with excitation at the base only is shown in fig.11.a. It can be observed that in all the three poses of the first link the path traced is same, but with excitation at the base and the joint 1, the trajectory of the manipulation depends on the initial pose of link 1 as shown in fig.11.b. Figure 11.c shows the manipulation with a combination of input joint velocities. In order to move the object further away the base joint must be excited with higher velocity.

Figure 12 illustrates the inverse kinematics. In fig.12.a the three configurations shown are 1) object beyond reach, 2) one point contact on link 3, and 3) three point contact. Fig.12.b illustrates 1)non-tangential contact with the feasible and infeasible regions of contact on the object, 2)two point contact configuration, and 3)situation where the object cuts into the base. Such a situation is classified as no contact for the purpose of simulations.

Figure 13 shows an inverse solution when a point on the object is specified as point of contact on link 3. Any point on link 3 can contact the object, however for

illustration three points on link 3 (base, midpoint, and tip) are considered. In fig.15, for mid point a mirror image of link 1 and link 2 is a possible solution. Link 3 can also be oriented in the opposite direction to the one shown in fig.15.

CONCLUSIONS

The analysis of the WAM shows that the manipulator can contact an object tangentially, non-tangentially or may not contact at all. The type of contact depends upon the manipulator and object dimensions and the location of the object. The animations illustrate both non-prehensile and prehensile manipulation. With single link manipulator the object is going away from the manipulator. The trajectory is an 'Archemedian' spiral. With 3-link manipulator it is possible to guide an object along a circular path. A 4-link manipulator can guide the object along a general trajectory by controlling the two input velocities. Conditions for bidirectional rotation and grasping are illustrated.

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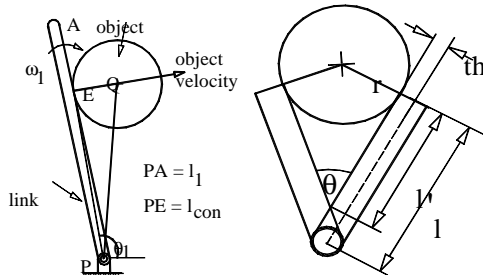


Fig. 1 One link manipulator

Fig. 2 Two link manipulator

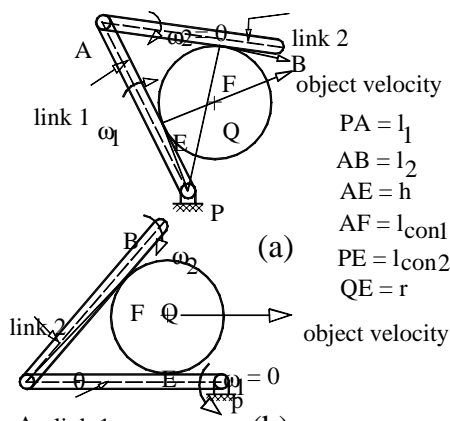


Fig. 3 Dyadic configuration

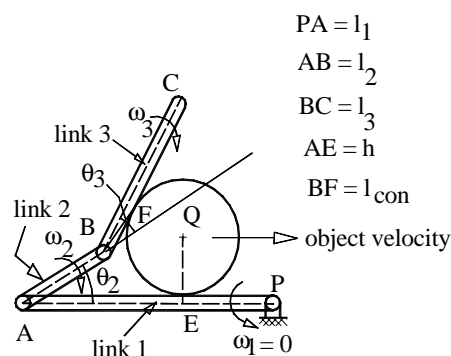


Fig. 4 Two points contact configuration

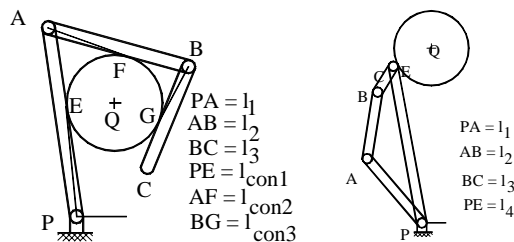


Fig. 5 Three points contact configuration

Fig. 6 Nontangential Contact

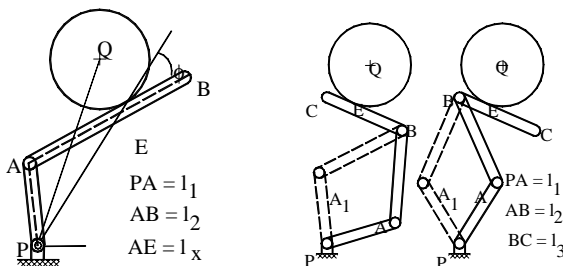


Fig. 7 Tangential contact on link 2

Fig. 8 Tangential contact on link 3

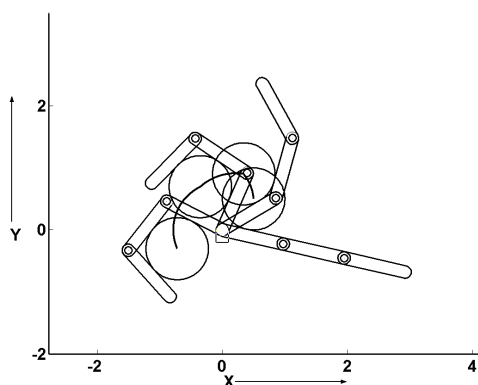


Fig. 9 Manipulation with 1,2,3 contact points

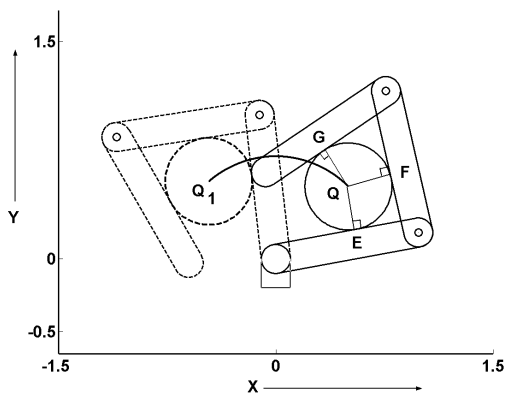


Fig. 10 Manipulation with 3 contact points (grasp is achieved)

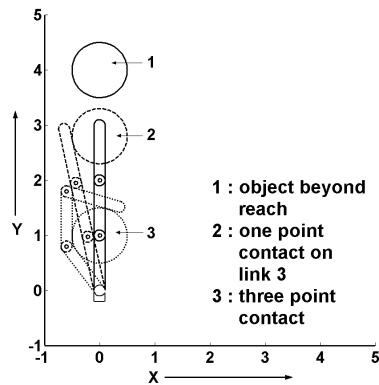


Fig. 12.a Inverse solutions

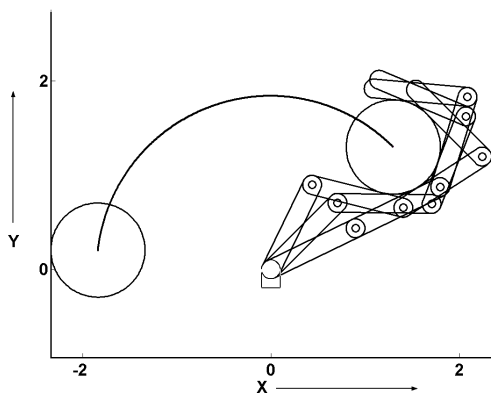


Fig. 11.a Four link manipulator ($\omega_2 = 0$)

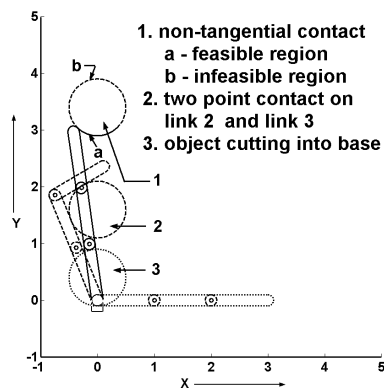


Fig. 12.b Inverse solutions (contd.)

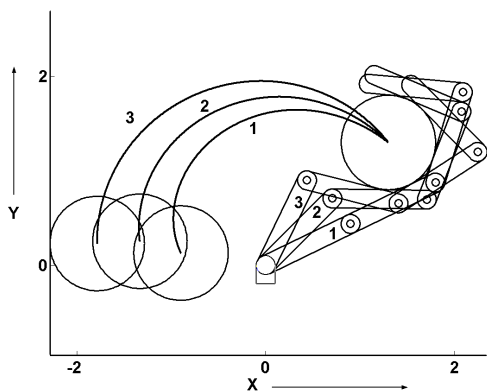


Fig. 11.b Four link manipulator ($\omega_1 = \omega_2$)

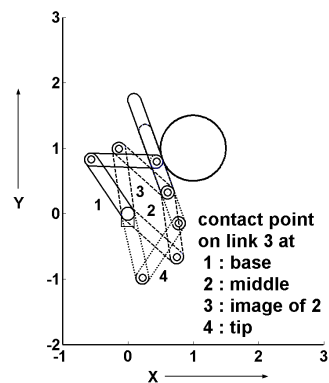


Fig. 13 Tangential contact on link 3 at a specified point

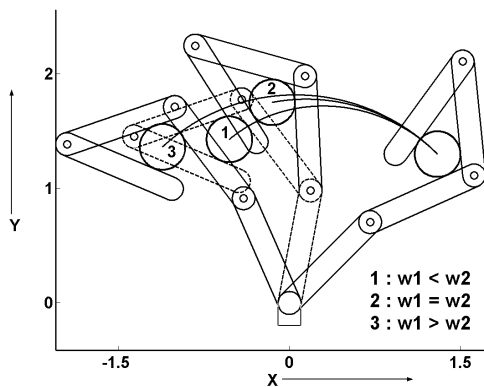


Fig. 11.c Four link manipulator